Midterm Review

Topic 1

- Define a trial, and random variables, and describe discrete/continuous probability distributions using PMF/PDF and CDFs

- Use probability rules to draw biological conclusions

- Describe one example of where each of the following distributions arises in ecology or evolution:
  - Discrete probability distributions
    - Poisson distribution
    - Bernoulli distribution
    - Binomial distribution
  - Continuous probability distributions
    - Normal distribution
    - Beta distribution
    - Exponential distribution
    - Gamma/Erlang distribution

- Define the moments and central moments of a probability distribution and derive the relationships between them. Use these definitions to draw biological conclusions using the distributions listed above.

- Sample randomly from any discrete and/or continuous probability distribution given its CDF.

Topic 2

- Describe different types of stochastic processes depending on their state and natural measurement of time. Give an example each from biology.

- Define the Markov property in discrete time

- Use a transition probability matrix to describe a discrete-time discrete-space Markov process (DTDS-MC)
    - List the properties of a transition rate matrix.

    - What is the definition of time homogenous process?

- Propose and justify a DTDS-MC model for a biological process.

- Characterize the states of a DTDS-MC as **transient**, **absorbing**, or **recurrent**. Use these mathematical characterizations of states to draw biological conclusions.

- Analyze a DTDS-MC and use these analyses to draw biological conclusions:
    - Use first-step analyses to find the absorption probabilities and time to absorption

    - Derive the stationary distribution

    - Numerical iterate a stochastic process

- Simulate DTDS-MC, calculate their moments through time and use these moments to draw biological conclusions

- Branching processes and their analysis
    - What is an example of a branching process?
    - Analyze branching processes using the probability of extinction

- Describe neutral genetic drift
    - What is the Wright-Fisher model?
    - What is the Moran model?

Topic 3

- Review the Exponential, Erlang, and Poisson distributions.
    - Show that the convolution of two exponential distributions (with the same rate) is an Erlang distribution with k=2.

- Define a Poisson process and describe:
    - The time to the next event
    - The time to the n^{th} event
    - The total number of events in time T
    - The superimposition of two Poisson processes
    - The thinning of two Poisson processes

- Propose a Poisson process describing a natural system and use it to draw biological conclusions
- Define a compound Poisson process and derive its first two moments.

- Simulate a Poisson or compound Poisson process.

- Use a Gillespie simulation to simulate a biological process

Topic 4

- What is the **Coalescent Process** and what does it describe?
    - What does the i^{th} coalescent time represent and what is its distribution?
    - What is the expected value of T_i?
    - What is the variance in T_i?
    - Draw an appropriately scaled coalescent genealogy

- What is the relationship between coalescent times and population size?

- Describe the assumptions of the infinite sites model of mutation
    - What are three measures of genetic diversity in this model of mutation?

    - Calculate the measures of genetic diversity from a given sample.

    - What is the expected number of segregating sites in a sample?

    - What is the expected number of pairwise differences in a sample?

- Simulate a coalescent process for a sample of size n with and without mutation.